

# On the logic of quantum physics and the concept of the time

Luigi Foschini

Institute TeSRE – CNR, Via Gobetti 101, I-40129, Bologna (Italy)

E-mail: [foschini@tesre.bo.cnr.it](mailto:foschini@tesre.bo.cnr.it)

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## Abstract

The logic-linguistic structure of quantum physics is analysed. The role of formal systems and interpretations in the representation of nature is investigated. The problems of decidability, completeness, and consistency can affect quantum physics in different ways. Bohr's complementarity is of great interest, because it is a contradictory proposition. We shall see that the flowing of time prevents the birth of contradictions in nature, because it makes a cut between two different, but complementary aspects of the reality.

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## 1 Introduction

The foundations of physics, the quantum logic, the interpretation of quantum mechanics: since the beginning, quantum theories gave rise to a broad variety of studies not properly defined studies devoted to settle the properties of this new branch of physics. Studies about quantum logic started in 1936, with the work of Birkhoff and von Neumann [3] and continued with many solutions, such as three-valued logics or unsharp approaches [2], [25]. However, several authors, even though with some differences, point out that

quantum mechanics does not require a new non-classical or unconventional logic [18], [30], [31]. Studies about the interpretation of quantum mechanics are quite plethoric [25], [50]. A widely accepted interpretation is the so called “Copenhagen interpretation”, initially developed by Niels Bohr and Werner Heisenberg, among others. However, the debate is still alive. There is no fundamental disagreement among physicists on how to use the theory for practical purposes, such as to compute energy levels, transition rates, etc. [35]. Problems arise on the concepts of quantum theory.

Many authors asked for a new language, different from the everyday one. Bohr stressed the difficulties of all means of expressing ourselves: human beings are “suspended in language”, as he used to say (see [37]). People strive to communicate experiences and ideas to others and to extend their field of descriptions, but this should be done in a way so that messages do not become too ambiguous. As far as the definition of experiment is concerned, Bohr said that it is a situation where it is possible to tell others what has been done and what has been learned and thus, observations results should be expressed in an unambiguous language [6].

The problem of the language of physics is not trivial. It could seem a negligible detail, because traditional philosophy regarded language as something secondary with respect to empirical reality. Still today there are many physicists believing that elementary particles are as real as an apple or a grapefruit can be (see [21]); Asher Peres emphasizes that experiments do not occur in a Hilbert space, but in a laboratory [35]. Many times one makes the mistake to search the meaning of a word or a symbol as something coexisting with the sign: there is a (con)fusion between *substance* and *substantive* [52].

The distinction between the mathematical symbol and the physical object is of paramount importance, because it allows to define what physics is. Mathematics is a language, invented by mankind, that we use to speak about nature and not *the* nature itself. Niels Bohr said that physics is simply what we can *say* about nature, not a way to find out how nature *is* (see [37]). There is no doubt about the existence of a world, but we must consider that no experience can be understood or communicated without a logic–linguistic frame. Both scholars and scientists elaborate ideas and report on the results. To make this they need of, at least, two languages: mothertongue and mathematics. We must learn to use them as any other laboratory instrument.

In this paper the reader does not find something of practical or predictive. We would like, in agreement with Bohr, to say something about nature by using the language of mathematics. A grain of knowledge, hoping that it

will be a *granum salis*. We have never seen something like the elaboration exposed in this paper and even though some things in the following sections may appear trivial, we think it is necessary to review them, in order to fix some definitions and notations at least. We hope it will be useful to see some old concepts in a different way. Moreover, we would like to point out a new possible definition of the concept of the time.

## 2 Formal systems

Before going on, let us recall some notions of formal systems. It is possible to distinguish between *formal logistic systems* and *interpreted languages*. In the first case, syntactical rules are sufficient to determine the system, in the second one semantic rules are also necessary. Generally speaking, a formal system is constituted by [22], [23], [28]:

1. a complete list of formal symbols or signs with which the expressions are formed;
2. an explicit, or also recursive, definition of what a well-defined formula is and what a term is;
3. a numerable set of well-defined formulas taken as axioms;
4. some metamathematical definitions, called rules of inference, which allow the transformation of formulas in other formulas or show what is an immediate consequence of a formula;
5. a list of formulas which can be derived with a finite number of applications of the rules of inference;
6. a list of sentences that allow us to shorten long expressions;
7. a list of sentences about the language that explicitly shows the denotation properties and, particularly, an indication of what expressions they denote and the circumstances in which an expression denotes a particular object.

Let us indicate the first type of formal system (formal logistic system or calculus) with the symbol  $FS(\vdash)$ , where  $\vdash$  is the well known symbol for formal

deduction.  $FS(\vdash)$  has the properties from 1 to 6 indicated above. We then indicate a formal system with semantics, or an interpreted language, with the symbol  $FS(\vdash, \models)$ , where  $\models$  is the symbol for the semantic deduction.  $FS(\vdash, \models)$  has the proprieties from 1 to 7 indicated above. The statement n. 7 is present for semantic systems only.

Now it is possible to construct several languages for various purposes, from everyday talking to science languages [8], [9]. It should be pointed out that there is a small difference between the list of mathematical logic and the alphabet of a language: in the second case, with a given interpretation, many of the formal symbols correspond to entire words and phrases rather than to a single letter [22]. The postulates and axioms for mathematics can be found in [22].

Statement n. 7, which makes the distinction between calculus and its interpretation, leads us to make a distinction between two types of truth, logic truth and factual one [9]. Truth in mathematics (a formal logistic system) is independent from the objects of the world and must be valid for every possible interpretation. Truth in physics (an interpreted language) is considered factual: a sentence is true when it corresponds to reality. This distinction is extremely important: in 1931, Tarski showed that it is possible to define truth only in purely formal languages [44]. Truth in an interpreted language cannot be defined inside the language itself. Tarski reported an useful example: the sentence “it is snowing” is true if and only if it is snowing. One must look out of the window whether it is snowing. This definition of truth must be extended according to Peirce, so that both for logic and factual truths it is possible to say that a proposition is true if, and only if, it is true what it *explicitly and implicitly* declares [33]. This is of paramount importance in science: it is thanks to this definitions that we can explore things outside our direct experience. For example, we can say that a black hole exists, even if we cannot directly observe it, because its existence follows from true propositions.

It is necessary to point out that mathematics is an interpreted language, even though the truth in this case is a logical one, because it is not possible to verify it with any external reality. A triangle is a geometric interpretation of a formal object in a certain formal logistic system, but it does not exist in reality. We can find objects with triangular shape, but not triangles.

Therefore, being physics an interpretation of the mathematics, we can conclude that it is an interpretation of an interpreted language. A semantics of a semantics. Objects in physics does not exists in reality, only as approx-

imations. In physics there is difference between a fluid and the water, in the same way as in mathematics there is difference between triangles and triangular-shaped objects.

In conclusion, we can always speak about truth in the language (interpreted or not), rather than truth with direct reference to the world.

### 3 Some features of formal systems

Main features of a formal system are decidability, completeness and consistency. As known, at the beginning of the XX century, the German mathematician David Hilbert presented a list of problems, among them there were [20]:

1. Decidability (*Entscheidungsproblem*), i.e. a formula  $A$  is decidable if  $\vdash A \vee \vdash \neg A$ .
2. Completeness (*Entscheidungsdefinitheit*), i.e. every closed formula of the formal system is decidable.
3. Consistency (*Widerspruchsfreiheit*), i.e. there are no contradictory formulas ( $\vdash A \wedge \vdash \neg A$ ).

Even though decidability is introductory to completeness, this problem was solved later, after the problems of completeness and consistency. Indeed, in 1931 Kurt Gödel solved problems 2 and 3 [17], while in 1936, Alan Turing [47] and Alonzo Church [12], independently solved question 1. Probably this gives the misleading impression that the *Entscheidungsproblem* is implied in Gödel's work, thus without the Church–Turing thesis.

Gödel demonstrated that in a formal system, like that set up by Whitehead and Russell in their book *Principia Mathematica* or like the Zermelo–Fraenkel–von Neumann axiom system of set theory, there are propositions undecidable on the basis of system's axioms, so that these formal systems cannot be complete (in the Gödel's paper this is stated in Theorem VI). This is valid for a wide class of formal systems, in particular for all systems that result from the two just mentioned with the addition of a finite number of axioms, *i.e.* those systems that fulfil the previously listed statements, from 1 to 6. As Gödel wrote, in a note added 28 August 1963, the characteristic property of a formal system is that reasoning in them, in principle, can be

completely replaced by mechanical devices [17]. Later on Tarski extended these results also to interpreted languages [45]. It is worth noting that the undecidability of a proposition can be skipped by stating an axiom that defines whether that proposition is true or false. However, even in such a new formal system there will be a new undecidable proposition. In order to have a complete formal system, we should have an infinite number of axioms.

Moreover, it is necessary to note that in this case we have implicitly assumed that the formal system is consistent. But, as Gödel himself showed in his famous paper, the consistency of the formal system is not provable in the formal system itself (this is stated in the Theorem XI) [17].

Gödel's results were later generalized by Church and Turing and used to solve the decision problem, which is closely connected to the problem of completeness, as we have already seen. The *Entscheidungsproblem* consists in finding a general algorithm, which would enable us to establish whether or not any particular sentence can be proved within that formal system. Church [12] and Turing [47] showed that this is not possible.

## 4 The logical structure of physics

Let us consider the logical structure of physics and the role of language. We recognize the existence of a reality, but to study it, to investigate it, to communicate our results, we need a language. The word does not change reality, but influences our way of thinking. It is necessary to analyze the relationship among words and things, physics and reality.

When children learn to talk, they correlate an object to a word: they give a meaning to a word and begin to believe that language is meaning-based. Only later they will know that language is syntax-based. For example, inserting the word ‘only’ into all possible positions in the sentence:

*I helped Mickey Mouse eat his cheese last week.*

it is possible to see that the meaning is a function of the syntax. During its “childhood”, physics was a correlation between an object and a mathematical symbol. It is then possible to speak about physics as interpretation of mathematical symbols, just like when we speak about the meaning of a word. Galileo Galilei wrote that physics is like a great book written in mathematical language, whose characters are triangles, circles, and other geometrical figures [16]. Later on, Charles Sanders Peirce founded the logic of relations,

noting that any deductive science establishes an isomorphism between the logic of nature and the logic of the language we employ to explore it [32].

A correlation is possible when objects can be found in the everyday life and everybody can see them. It is possible to make a correlation between an apple and the word ‘apple’. Anyone can see, touch, smell, eat or hear (when it falls down) an apple: we can use our five senses to know the apple. Therefore, there is no doubt about the meaning of the word ‘apple’; moreover it must be stressed that there is a correlation only and not a (con)fusion between an apple and the word ‘apple’.

The correlation is not so clear when we speak, for example, about electromagnetism. Nobody can see, touch, smell, eat or hear a radio wave: it is necessary to use an instrument, the radio receiver, in order to hear radio waves. The use of analogies based on everyday life can be misleading, as known from the hydrodynamic model of electric current.

Things become more complicated when we speak about quantum mechanics. The exigence of observing a world outside our direct experience and with instruments that must be within human beings’ reach, forces us to emphasize the experiment procedure [35]. Only certain things are observable and this happens only after an irreversible act of amplification, such as the blackening of a grain of silver bromide emulsion or the triggering of a photodetector: what we choose to measure has an unavoidable consequence on what we shall find [49]. It is possible to correlate a mathematical symbol only to those “observables”, *i.e.* only observables can have a meaning. All other mathematical symbols or words used in quantum mechanics do not have any meaning. The exigence of abstraction must be emphasized.

Let us consider an example: the d’Alambert wave equation, which can be written as following:

$$\frac{\partial^2 \psi(\mathbf{r}, t)}{\partial t^2} - c^2 \nabla^2 \psi(\mathbf{r}, t) = 0 \quad (1)$$

This equation shows that there is a logical link among the symbols indicated. This link is formal and then independent from meaning. Then, Eq. (1) can be used in several branches in physics, according to the meaning that one gives to used symbols. The function  $\psi$  can be correlated, even though without a one-to-one correspondence, with pressure, velocity, velocity potential, displacement of matter points from equilibrium in a solid, electric field, magnetic induction or the corresponding potentials. Moreover there are many other interpretations of Eq. (1) (for examples, see [26]). The interpretation

of other symbols follows as a consequence of the choice of interpretation of  $\psi$ . It should be noted that the interpretation does not directly concern the Eq. (1), but through the boundary conditions or through the initial values. However, it should be stressed that the wide application field of the wave equation does not derive from its physical interpretation. Starting with an interpreted wave equation one can obtain misleading results. For example, one cannot consider pressure waves analogous to electromagnetic waves: in the first case there are some consequences absent in the second one (*e.g.* the supersonic “bang”). The formal character of physics, and of any deductive science, is due to the fact that when deducing a theorem from the postulates, it is necessary to avoid any specific property of the interpretation and to use only those formal properties which are explicitly stated in postulates (and therefore, belong to every interpretation of the formal system) [46]. The final conclusion is that every theorem of a given deductive theory is satisfied by any interpretation of the formal system of this theory [46].

Let us now consider the Schrödinger wave equation:

$$i\hbar \frac{\partial \psi(\mathbf{r}, t)}{\partial t} + \frac{\hbar^2}{2m} \nabla^2 \psi(\mathbf{r}, t) - U\psi(\mathbf{r}, t) = 0 \quad (2)$$

It must be noted that  $\psi$ , at this stage, has no meaning and it is not possible to find anyone. As known, it is necessary to make further operations on  $\psi$ . For example, the probability  $dP$  to find an electron in a volume  $dV$  centered in the point  $\mathbf{r}$  is:

$$dP = |\psi(\mathbf{r})|^2 dV \quad (3)$$

There is no strict correlation between the symbol and the object. It should be underlined that experiments are always macroscopic, because they must be readable by a human being. We cannot see an electron directly, but we can observe some effect on some instrument. This does not involve any particular logic or physical law, but a strong attention to the interpretation.

The question of the correlation between a physical object and a mathematical symbol in quantum theory gave rise to a wide debate. The most famous one occurred between Albert Einstein and Niels Bohr and is synthetized in the EPR paradox [5], [14]. Already in the initial words of the Einstein, Podolski and Rosen’s paper [14] it is possible to see the authors’ conception the physics: each element of reality corresponds to an element in the theory. This type of physics is strongly based on the meaning of a word or

a symbol. Bohr's reply shows that this conception is ambiguous and then, it can generate paradoxes [5]. Bohr underlined that quantum mechanics forces us to a radical revision of the traditional idea of an absolute physical reality, as Einstein's relativity led us to abandon the idea of an absolute space. In each experiment, owing to the impossibility of discrimination, between who is the observer and what is observed, the experiment became simply a situation where we can tell others what we have done (see also [4], [6]). Experimental procedures must be strictly specified in order to minimize ambiguities.

Physics is then an interpreted language that we use in order to say something about nature (see [37]) and the syntax of this language is isomorphic to the logic of nature. Physics is constructed, as every language, following the rules listed in Section 2. We can then write that classical physics is a semantic system,  $FS(\vdash, \models)$ , where only the experiment can tell if a sentence is true. The system must be consistent, that is each provable formula must be valid. It is possible to write that a formula  $A$  is said to be true when:

$$if \vdash A \text{ then } \models A \quad (4)$$

that is the *consistency condition*. Eq. (4) says that this conditional statement is an implication, that is  $\models A$  is true under all conditions for which  $\vdash A$  is provable. It is also possible to say that the provability of  $\vdash A$  "forces"  $\models A$  to be true.

On the other hand, the *completeness condition* says that each valid formula is provable. By using symbols:

$$if \models A \text{ then } \vdash A \quad (5)$$

Realists think that only Eq. (5) is valid in physics. When the semantic deduction is absent, as in quantum mechanics, realists force the existence of  $\models A$ , according to their own logic. In doing this, realists find paradoxes, that, however, are in assuming their own logic as the semantic deduction isomorphic to the one of the nature.

Indeed, in quantum mechanics, the main problem is the measurement, that is to find a meaning for mathematical symbols. First of all, it is worth noting that the existence of the Heisenberg's indeterminacy principle restricts the variables that can be measured simultaneously [19]. It should be noted that even in classical physics it is not possible to measure two variables, or more, *simultaneously*, owing to experimental uncertainties; nevertheless, taking into account the smallness of these uncertainties with respect to macro-

scopic variables, it is possible to assume that the measurement are *simultaneous*. In quantum mechanics, in addition to experimental uncertainties, there is also Heisenberg’s principle, that allows to measure canonically conjugate quantities simultaneously only with a characteristic indeterminacy.

This feature called for a new concept of measurement, developed and analyzed by several authors during this century [6], [7], [35], [48]; see also in [50]. However, Bohr [6] (see also [37]) and, later, Peres [35], wrote against the existence of a “quantum measurement”; Bohr designed, in a semi-serious style, a series of devices that should serve to make these measurements [6].

It is necessary to stress that an experiment is always macroscopic, in order to be accessible to a human being. We never directly observe a microscopic variable: nobody has very *seen* a single photon, but only some *macroscopic irreversible effects*. In order to explain these effects, we use quantum mechanics, from which we can reconstruct a visualization of a process outside our direct experience.

The postulates of quantum mechanics stated by von Neumann [48] are often considered as providing an interpretation in themselves, but it is necessary to restrict the notion of interpretation and to give a meaning only to those words or symbols which have referents, that are always macroscopic in order to be reachable by human beings. These axioms and postulates are widely dealt in those books and we refer to them for readers interested in further investigations.

Having a formal system, we can apply Gödel’s procedure and make the arithmetization of the quantum theory. To do this, we must assign an integer to a sequence of signs or to a sequence of sequences of signs. For example, it is possible to use the number 5 instead of the word ‘not’ or 9 instead of ‘for all’, and so on. Notions such as ‘formula’ and ‘provable formula’ can now be represented by integers or sequences of integers. This method is today commonly used: in a computer we represent all concepts by using sequences of 0 and 1 only. Indeed, Turing used Gödel’s numbers in order to construct his famous machine, that is the logical basis of modern computers [47].

Now it is not necessary to repeat the proof of incompleteness: it is sufficient to have shown that quantum theory is a formal system and then incompleteness logically follows, as shown by Gödel. The extension of incompleteness to physics is quite straightforward, as underlined by Svozil by means of other techniques [43].

## 5 Complementarity, Indeterminacy, Contradictions

Many scientists tried to see in physics reminiscences or analogies with Gödel's theorems: Peres and Zurek looked for analogies with quantum theory [34], [36]; Svozil extended incompleteness theorems to physics by using Turing's proof [43]; Komar investigated the quantum field theory [24]; Casti examined the interconnections between chaos and Gödel's theorem [10] and last, but not least, Chaitin analysed incompleteness in information theory [11]. It is worth to note the works by Chaitin [11] and Svozil's [43]: they suggested that incompleteness was natural and widespread rather than pathological and unusual. Particularly, Svozil extended undecidability to physics, by means of the Turing's proof [43].

Scientists often connected undecidability with Heisenberg's indeterminacy (uncertainty) in physics. However, undecidability does not mean uncertainty, even though they can be linked somehow. It is possible to demonstrate that uncertainty cannot be undecidable: for example, let us consider the uncertainty relation for position and momentum:

$$\Delta x \Delta p \geq \frac{\hbar}{2} \quad (6)$$

This proposition holds, while the negation of the uncertainty principle, *i.e.*:

$$\Delta x \Delta p = 0 \quad (7)$$

does not hold. We recall that, a formula  $A$  is said to be undecidable when it is not possible to prove nor  $A$  neither  $\neg A$ . For uncertainty we can prove Eq. (6), but we cannot prove Eq. (7), where Eq. (7)=  $\neg$  Eq. (6). *Then, uncertainty is not an undecidable proposition.*

Other scientists believe that physics cannot be subjected to Gödel's theorems because it has variables of a higher order, such as experimental data, but Tarski has already shown that this is not sufficient [45]. Morevoer, Svozil recently showed that undecidability is present in physics, so that it cannot be complete [43].

We want now to explore the consistency of quantum physics and a particular feature, Bohr's complementarity, taking into account the logic–linguistic analysis carried out until now. It is necessary to analyze also some implicit

postulates, holding in physics: for example, all propositions must be consistent. As a consequence of Gödel's Theorem XI (it is not possible to prove the consistency of a formal system within the system), condition (4) must be assumed as an axiom:

Axiom 1: *if*  $\vdash A$  *then*  $\models A$

Another implicit axiom is that each event is unique (*i.e.*, there is only a single world). As we know from Heisenberg's indeterminacy, it is not possible to measure simultaneously position and momentum, while Bohr's complementarity tells us that, even though particle and wave behaviour are mutually exclusive, they are also complementary [4]. There is no well-defined proposition that states what complementarity is. Bohr wrote:

The very nature of the quantum theory thus forces us to regard the space-time co-ordination and the claim of causality, the union of which characterizes the classical theories, as complementary but exclusive features of the description, symbolizing the idealization of observation and definition respectively. [...] we learn from quantum theory that the appropriateness of our usual causal space-time description depends entirely upon the small value of the quantum of action as compared to the actions involved in ordinary sense perceptions [4].

We must stress the importance of time in this quotation, that Bohr recalled also in his reply to Einstein, Podolsky and Rosen [5]. The implicit postulate of uniqueness of each event must be reformulated in:

Axiom 2: *There is a single world at a time  $t$ .*

Already at first sight, Bohr's complementarity can be seen as a contradictory proposition because, for example, an electron cannot behave simultaneously as a particle or as a wave. In the famous experiment of the double slit, we can observe interference (wave) if there is no specification of which path (particle). But if the path is specified, then the interference will disappears. During these last years, there was a vivid debate around complementarity: it started with some experiments carried out by Scully *et al.* [39], where they claimed to have observed the simultaneous behaviour of waves and particles, avoiding the indeterminacy principle. This episode could be dramatic for

physics, because a simultaneous observation of wave and particle behaviour would contradict the implicit Axiom 2. Some years later, Storey *et al.* replied that the Heisenberg's principle is valid and then Scully *et al.* were wrong [41]. This was followed by a letter of Englert *et al.* [15], to which Storey *et al.* soon reply [42]. Moreover, Wiseman and Harrison wrote a letter in which they claimed that both groups were right [51].

It is worth to note that in all these discussions no adequate attention was given to time. The indeterminacy principle says that, at a given time  $t$ , we can observe, with sufficient precision, one canonically conjugate variable only: position or momentum, particle or wave. When we prepare an experiment, we choose what we want to observe, particle or wave. This is equal to select a particular interpretation (semantic deduction) for the formal system. Let us consider these two propositions:

- $A = \text{"an electron behave like a wave"}$
- $B = \text{"an electron behave like a particle"}$

which are complementary, as known. Owing to the fact that at a time  $t$  we must have a single world, it is possible to write that:

$$B = \neg A \text{ (or } A = \neg B\text{)} \quad (8)$$

Let us suppose that we set up an experiment to observe, say, the particle behaviour of an electron. So  $B$  is true, while  $A = \neg B$  is false. *In a second time*, we can set up a new experiment to detect the wave behaviour of the electron. In this case,  $A$  is true, while  $B = \neg A$  is false. If we want to construct a formal system for quantum mechanics we should select one of these behaviours, but it is not possible to set up an experiment that allows to decide whether  $A$  or  $B$  is always true. We have that both can be true and false: this is a contradiction.

Time is the only thing that prevents the birth of a contradiction in nature. The flowing of time makes the existence of two different aspects possible, avoiding the contradiction of a simultaneous existence (intuitively we could see time as a "cut" in space). Neglecting time in quantum mechanics could then lead to contradictory propositions, because both  $A$  and  $\neg A$  (or  $B$  and  $\neg B$ ) simultaneously hold and this contradicts Axiom 2. Time creates a cut between two different, but complementary aspects of reality. It is also possible to deduce that we cannot unificate physics under a single description: we need both waves and particles.

## 6 Time in quantum mechanics

“How much time”, “It took quite some time”, “There is plenty of time”. These are some examples only of our common way to think about time: an interval between two instants. In physics as well time is seen as an interval. Asher Peres stressed that the measurement of time is the observation of a dynamical variable, which law of motion is known and it is uniform and constant in time [35]. There is a kind of self-reference in this definition and nothing is said about time. On the other hand, time is considered simply as a parameter and, according to this, the above definition is completely satisfactory. Moreover, time is sometimes neglected (e.g. steady state phenomena), which is useful to understand some physical concepts.

However, when we deal with quantum mechanics the problem of the time explodes in all its complexity. In classical mechanics (hamiltonian formulation) the dynamical state of a physical system is described by a point in a phase space, that is we have to know position  $q$  and momentum  $p$  at a given time  $t$ . Even though it can appear a sophism, it is not possible, strictly speaking, to know simultaneously  $q$  and  $p$  of any object. However, in classical physics, we may neglect variations during the lapse of time between the measurement of  $q$  and  $p$ , because the quantum of action is so small when compared to macroscopic actions. It is very interesting to note how Sommerfeld stressed this question when he wrote about the Hamilton’s principle of least action: the trajectory points  $q$  and  $q + \delta q$  are considered *at the same time instant* [40].

In quantum mechanics this approximation is not valid, because actions are comparable with the quantum of action. The hamiltonian formalism is not anymore a useful language to investigate nature and, as known, it was necessary to settle quantum mechanics.

The impossibility to neglect time in quantum mechanics is well described by the Heisenberg’s principle of indeterminacy. Nevertheless, the role of the time in quantum indeterminacy is often neglected. In the history of physics, we can often find authors which claimed to have found a way to avoid the obstacle of indeterminacy. However they all missed the target, that is the question of the time. Heisenberg clearly stated that indeterminacy relationships do not allow a simultaneous measurement of  $q$  and  $p$ , while do not prevent from measuring  $q$  and  $p$  taken in isolation [19]. It is possible to measure, with great precision, complementary observables in two different time instants: this is not forbidden by Heisenberg’s principle. Later on, it

is also possible to reconstruct one of the observables at the reference time of the other observable, but this is questionable. In the interval between two measurements observables change because time flows. What happened during this interval? We can reconstruct observables by making hypotheses, but we have to remember that these are hypotheses and not measurements.

We have to take into account the so-called “energy–time uncertainty relationship”. As known, time is a *c*–number and therefore it have to commute with each operator. Nevertheless, the relationship exists, but it is worth to note its dynamical nature, whereas indeterminacy is kinematic [1]. That is, it follows from the evolution of the system during the measurement. Bohr had already stated this and he had often pointed out the time issue [4] [5], along with Landau and Peierls [27]. We refer to [27] in which the question is stated in a better way. The relationship:

$$\Delta E \Delta t > \hbar \quad (9)$$

means that we have to consider the system evolution during the measurement, that is the difference between the measurement result and the state after the measurement. The energy difference between the two states cannot be less than  $\hbar/\Delta t$ . The energy–time relationship has important consequences particularly as regards the momentum measurement and, therefore, on double–slit experiment [27].

Eq. (9) suggests that, given a certain energy, it is possible to construct a state with a huge  $\Delta E$  in order to obtain a very small  $\Delta t$ . However, in a recent paper, Margolus and Levitin [29] give a strict bound that depends on the difference between the average energy of the system and its ground state energy. Is it a step toward a quantization of the time?

In addition, if we consider the equation of motion (written with Dirac’s notation [13]):

$$i\hbar \frac{d|Pt>}{dt} = H(t)|Pt> \quad (10)$$

we can see that  $H(t)$  is  $i\hbar$  times an operator of time–translation. If the system is closed we can consider  $H$  constant and equal to the total energy of the system; but if not, if energy depends on time, this means that the system is under the action of external forces (e.g. measurement). The measurement introduce an energy exchange that does not follow causality.

Moreover, it is worth to note that a closed system is an abstraction. A real closed system is not observable, without introducing energy exchange

which would change  $H$ : therefore that would not be a closed system. We can say, by means of Rovelli's words that there is no way to get information about a system without physically interacting with it for a certain time [38].

Would you consider it a sophism? Of course not. We should always bear in mind that quantum physics is only an interpreted language we use to speak about Nature, though it does not describe Nature itself. In classical physics we made many approximations, which are no longer valid in quantum physics. In particular, we can no more neglect time. As Heraclitus stated, you cannot plunge your hands twice in the same stream.

## 7 Conclusions

In this paper, the formal character of quantum mechanics is emphasized, showing the clear distinction between mathematics and nature, words and objects. As Bohr used to say, physics concerns what we can say about nature, by using the language of mathematics. The interpretation of quantum mechanics, that is a correlation between a symbol and an object, is only a limit process, because every experiment is macroscopic, in order to be reached to human beings. Owing to its formal character, quantum theory is subjected to Gödel's incompleteness theorems.

Bohr's complementarity can be a useful tool to investigate time, because emphasize that the flowing of time prevent the birth of contradictions in nature. Time makes a cut between two different, but complementary aspects of reality. The link between complementarity and time requires further investigations.

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## 9 List of symbols

- $\vdash$ : formal deduction;

- $\models$ : semantic deduction;
- $\neg A$ : not- $A$ ;
- $\vee$ : or;
- $\wedge$ : and.

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